Mark Scheme Q1.

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = v + x\frac{dv}{dx}$ seen	B1
	$3x^3v^2\left(v + x\frac{dv}{dx}\right) = x^3 + v^3x^3 \quad \Rightarrow \quad 3v^2x\frac{dv}{dx} = 1 - 2v^3$ (**ag**)	M1 A1 cso
		(3)
(b)	$\int \frac{3v^2}{1-2v^3}\,dv = \int \frac{1}{x}\,dx$	M1
	$-\frac{1}{2}\ln(1-2v^3) = \ln x \ (+C)$	M1 A1
	$-\ln(1-2v^3) = \ln x^2 + \ln A$	
	$Ax^2 = \frac{1}{1-2v^3}$	M1
	$1 - \frac{2y^3}{x^3} = \frac{1}{Ax^2}$	
	$y = \sqrt[3]{\frac{x^3 - Bx}{2}}$ or equivalent	dM1 A1cso
		(6)
(c)	Using $y = 2$ at $x = 1$: $12\frac{dy}{dx} = 1 + 8$	M1
	At $x = 1$, $\frac{dy}{dx} = \frac{3}{4}$	A1
		(2) **11**
	Notes	
(a)	M1 for substituting y and $\frac{dy}{dx}$ obtaining an expression in v and x only	
(b)	1st M1 for separating variables 2nd M1 for attempting to integrate both sides 1st A1both sides required or equivalent expressions. (Modulus not required.) 3rd M1 Removing logs, dealing correctly with constant 4th M1 dep on 1st M. Substitute $v = \frac{y}{x}$ and rearranging to $y = f(x)$	
(c)	M1 for finding a numerical value for $\frac{dy}{dx}$ A1 for correct numerical answer oe.	

Q2.

Question Number	Scheme		Marks
	Way 1		
(a)	$v = y^{-3} \Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}y} = -3y^{-4}$	Correct derivative	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}v}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\dfrac{y^4}{3}\dfrac{\mathrm{d}v}{\mathrm{d}x}$ Or $-3y^{-4}\dfrac{\mathrm{d}y}{\mathrm{d}x}x - 3y^{-3} = -6x^4$	M1: Correct use of the chain rule A1: Correct equation	M1A1
	$-\dfrac{y^4}{3}\dfrac{\mathrm{d}v}{\mathrm{d}x}x + y = 2x^4y^4$		
	$-\dfrac{y^4}{3}\dfrac{\mathrm{d}v}{\mathrm{d}x}x + y = 2x^4y^4 \Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}x} - \dfrac{3v}{x} = -6x^3$	dM1: Substitutes to obtain an equation in v and x. A1: Correct completion with no errors seen	dM1A1
	Way 2		
	$y = v^{-\frac{1}{3}} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}v} = -\frac{1}{3}v^{-\frac{4}{3}}$	Correct derivative	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}v}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -\frac{1}{3}v^{-\frac{4}{3}}\dfrac{\mathrm{d}v}{\mathrm{d}x}$	M1: Correct use of the chain rule A1: Correct equation	M1A1
	$-\dfrac{v^{-\frac{4}{3}}}{3}\dfrac{\mathrm{d}v}{\mathrm{d}x}x + v^{-\frac{1}{3}} = 2x^4v^{-\frac{4}{3}}$	dM1: Substitutes to obtain an equation in v and x.	dM1
	$-\dfrac{v^{-\frac{4}{3}}}{3}\dfrac{\mathrm{d}v}{\mathrm{d}x}x + v^{-\frac{1}{3}} = 2x^4v^{-\frac{4}{3}} \Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}x} - \dfrac{3v}{x} = -6x^3$	A1: Correct completion with no errors seen	A1
	Way 3 (Working in reverse)		
	$v = y^{-3} \Rightarrow \dfrac{\mathrm{d}v}{\mathrm{d}y} = -3y^{-4}$	B1: Correct derivative	B1
	$\dfrac{\mathrm{d}v}{\mathrm{d}x} = \dfrac{\mathrm{d}v}{\mathrm{d}y}\dfrac{\mathrm{d}y}{\mathrm{d}x} = -3y^{-4}\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1: Correct use of chain rule A1: Correct expression for $\mathrm{d}v/\mathrm{d}x$	M1A1
	$-3y^{-4}\dfrac{\mathrm{d}y}{\mathrm{d}x} - \dfrac{3y^{-3}}{x} = -6x^3$	M1: Substitutes correctly for $\dfrac{\mathrm{d}v}{\mathrm{d}x}$ and v in equation (II) to obtain a D.E. in terms of x and y only. A1: Correct completion to obtain equation (I) with no errors seen	dM1A1

Question Number	Scheme		Marks
(b)	$I = \mathrm{e}^{\int -\frac{3}{x}\mathrm{d}x} = \mathrm{e}^{-3\ln x} = \frac{1}{x^3}$	M1: $\mathrm{e}^{\int \pm\frac{3}{x}\mathrm{d}x}$ and attempt integration. If not correct, $\ln x$ must be seen. A1: $\frac{1}{x^3}$	M1A1
	$\frac{v}{x^3} = \int -6\,\mathrm{d}x = -6x(+c)$	M1: $v \times$ their $I = \int -6x^3 \times$ their $I\,\mathrm{d}x$ A1: Correct equation with or without $+c$	dM1A1
	$\frac{1}{y^3x^3} = -6x + c \Rightarrow y^3 = ...$	Include the constant, then substitute for y and attempt to rearrange to $y^3 = ...$ or $y = ...$ with the constant treated correctly	ddM1 dep on both M marks of (b)
	$y^3 = \frac{1}{cx^3 - 6x^4}$	Or equivalent	A1 (6) **Total 11**

Q3.

Question Number	Scheme	Notes	Marks
(i)			
NB	If candidates appear to be considering any/all of p, q, r to be non-positive, send the attempt to review.		
	By use of integrating factor:		
(i) (a)	$\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{q}{p}x = \frac{r}{p}$		
	$\mathrm{e}^{\int \frac{q}{p}dt} = \mathrm{e}^{\frac{qt}{p}}$		
	$x\mathrm{e}^{\frac{qt}{p}} = \int \frac{r}{p}\mathrm{e}^{\frac{qt}{p}}\; dt$	Obtain IF $\mathrm{e}^{\pm\int \frac{q}{p}dt} = \mathrm{e}^{\pm\frac{qt}{p}}$, multiply through by it and integrate LHS. Accept $\int r\mathrm{e}^{\pm\frac{qt}{p}}\mathrm{d}t$ for RHS	M1
	$x\mathrm{e}^{\frac{qt}{p}} = \frac{r}{q}\mathrm{e}^{\frac{qt}{p}}(+c)$	Integrate RHS $\mathrm{e}^{\frac{qt}{p}} \to k\mathrm{e}^{\frac{qt}{p}}$. Constant of integration may be missing. Dependent on the first M mark.	dM1
	$t = 0, x = 0, c = -\frac{r}{q}$	Substitute $x = 0$ and $t = 0$ to obtain c. Dependent on both M marks above.	ddM1
	$x\mathrm{e}^{\frac{qt}{p}} = \frac{r}{q}e^{\frac{qt}{p}} - \frac{r}{q}$		
	$x = \frac{r}{q} - \frac{r}{q}e^{-\frac{q}{p}t}$	oe Change to $x = ...$	A1

ALT:	By separating the variables:		
(i) (a)	$\int \frac{p\,\mathrm{d}x}{r-qx} = \int \mathrm{d}t$	Attempt to separate variables	M1
	$-\frac{p}{q}\ln(r-qx) = t\ (+c)$	Integrate to give ln Constant of integration may be missing.	dM1
	Use $t=0, x=0$	Substitute $x=0$ and $t=0$ to obtain their constant.	ddM1
	$x = \frac{r}{q} - \frac{r}{q}e^{-\frac{q}{p}t}$	Oe	A1
			(4)
(b)	$t\to\infty,\ e^{-\frac{q}{p}t}\to 0,$		
	$(x\to)\frac{r}{q}$	Cao	B1
			(1)

(ii)	$ye^{2\theta} = \int e^{2\theta}\sin\theta\ \mathrm{d}\theta$	Multiply through by IF of the form $e^{\pm 2\theta}$ and integrate LHS (RHS to have integral sign or be integrated later). IF = $e^{2\theta}$ and all correct so far.	M1 A1
	$ye^{2\theta} = \left[-e^{2\theta}\cos\theta\right] + 2\int e^{2\theta}\cos\theta\,\mathrm{d}\theta$ Or $\left[\frac{1}{2}e^{2\theta}\sin\theta\right] - \frac{1}{2}\int e^{2\theta}\cos\theta\,\mathrm{d}\theta$	Use integration by parts once (signs may be wrong)	M1
$\left(ye^{2\theta}=\right)$	$\left[-e^{2\theta}\cos\theta\right] + 2\left\{\left[e^{2\theta}\sin\theta\right] - 2\int e^{2\theta}\sin\theta\,\mathrm{d}\theta\right\}$ Or $\frac{1}{2}e^{2\theta}\sin\theta - \frac{1}{2}\left[\frac{1}{2}e^{2\theta}\cos\theta + \frac{1}{2}\int e^{2\theta}\sin\theta\,\mathrm{d}\theta\right]$	Use parts a second time (Sim conditions to previous use) Must progress the problem - not just undo the first application	M1
	$\left(ye^{2\theta}=\right) - e^{2\theta}\cos\theta + 2e^{2\theta}\sin\theta - 4\int e^{2\theta}\sin\theta\,\mathrm{d}\theta$ Or $\frac{1}{2}e^{2\theta}\sin\theta - \frac{1}{4}e^{2\theta}\cos\theta - \frac{1}{4}\int e^{2\theta}\sin\theta\,\mathrm{d}\theta$	RHS correct	A1
	$ye^{2\theta} = -e^{2\theta}\cos\theta + 2e^{2\theta}\sin\theta - 4ye^{2\theta} + c$ Or $ye^{2\theta} = \frac{1}{2}e^{2\theta}\sin\theta - \frac{1}{4}e^{2\theta}\cos\theta - \frac{1}{4}ye^{2\theta} + c$ $ye^{2\theta} = \int e^{2\theta}\sin\theta\ \mathrm{d}\theta = \frac{1}{5}e^{2\theta}(2\sin\theta - \cos\theta)(+c)$ $\theta=0, y=0 \Rightarrow C = \frac{1}{5}$	Replaces integral on RHS with integral on LHS (can be $ye^{2\theta}$ or $\int e^{2\theta}\sin\theta\ \mathrm{d}\theta$) **and** uses $\theta=0, y=0$ to obtain a value for the constant. Depends on the second M mark	dM1
	$y = \frac{1}{5}(2\sin\theta - \cos\theta) + \frac{1}{5}e^{-2\theta}$	oe	A1cso (7)

ALT:	By aux equation method:		
	$m+2=0 \Rightarrow m=-2$	Attempt to solve aux eqn	M1
	CF $(y=)Ce^{-2\theta}$	oe	A1
	PI $(y=)\ \alpha\sin\theta+\beta\cos\theta$	PI of form shown oe	M1
	$\frac{dy}{d\theta}=\alpha\cos\theta-\beta\sin\theta$		
	$\alpha\cos\theta-\beta\sin\theta+2\alpha\sin\theta+2\beta\cos\theta=\sin\theta$	Diff and subst into equation	M1
	$2\alpha-\beta=1, \alpha+2\beta=0 \Rightarrow \alpha=\frac{2}{5}, \beta=-\frac{1}{5}$	Both $\alpha=\frac{2}{5}, \beta=-\frac{1}{5}$	A1
	$\theta=0, y=0 \Rightarrow C=\frac{1}{5}$	Use $\theta=0, y=0$ to obtain a value for the constant	dM1
	$y=\frac{1}{5}(2\sin\theta-\cos\theta)+\frac{1}{5}e^{-2\theta}$	Must start $y=\ldots$.	A1cso(7) **Total 12**
NB	If the equation is differentiated to give a second order equation and an attempted solution seen – send to review.		

Q4.

Question Number	Scheme	Marks
	$\frac{dy}{dx}+5\frac{y}{x}=\frac{\ln x}{x^2}$ Integrating factor $e^{\int\frac{5}{x}}$	M1
	$e^{\int\frac{5}{x}}=e^{5\ln x}=x^5$	A1
	$\int x^3\ln x\,dx=\frac{x^4\ln x}{4}-\int\frac{x^3}{4}dx$	M1 M1 A1
	$=\frac{x^4\ln x}{4}-\frac{x^4}{16}\ (+C)$	A1
	$x^5y=\frac{x^4\ln x}{4}-\frac{x^4}{16}+C$ $\qquad y=\frac{\ln x}{4x}-\frac{1}{16x}+\frac{C}{x^5}$	M1 A1
		(8) **8**
	1st M1 for attempt at correct Integrating Factor 1st A1 for simplified IF 2nd M1 for $\frac{\ln x}{x^2}$ times their IF to give their '$x^3\ln x$' 3rd M1 for attempt at correct Integration by Parts 2nd A1 for both terms correct 3rd A1 constant not required 4th M1 $x^5y=$ their answer $+C$	

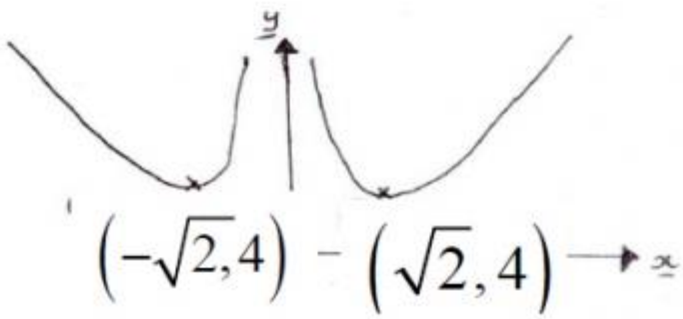
Q5.

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = \frac{dy}{dz}.\frac{dz}{dx}$ and $\frac{dy}{dz} = 2z$ so $\frac{dy}{dx} = 2z.\frac{dz}{dx}$	M1 M1 A1
	Substituting to get $2z.\frac{dz}{dx} - 4z^2 \tan x = 2z$ and thus $\frac{dz}{dx} - 2z \tan x = 1$ *	M1 A1 (5)
(b)	$\text{I.F.} = e^{\int -2\tan x dx} = e^{2\ln\cos x} = \cos^2 x$	M1 A1
	$\therefore \frac{d}{dx}\left(z\cos^2 x\right) = \cos^2 x \quad \therefore z\cos^2 x = \int \cos^2 x dx$	M1
	$\therefore z\cos^2 x = \int \frac{1}{2}(\cos 2x + 1)\, dx = \frac{1}{4}\sin 2x + \frac{1}{2}x + c$	M1 A1
	$\therefore z = \frac{1}{2}\tan x + \frac{1}{2}x\sec^2 x + c\sec^2 x$	A1 (6)
(c)	$\therefore y = (\frac{1}{2}\tan x + \frac{1}{2}x\sec^2 x + c\sec^2 x)^2$	B1ft (1) **12**

Q6.

Question Number	Scheme		Marks
Q	$\sin x \dfrac{dy}{dx} - y\cos x = \sin 2x \sin x$		
	$\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin x} = \dfrac{\sin 2x \sin x}{\sin x}$	An attempt to divide every term in the differential equation by $\sin x$. Can be implied.	M1
	$\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin x} = \sin 2x$		
	Integrating factor $= e^{\int -\frac{\cos x}{\sin x} dx} = e^{-\ln \sin x}$	$e^{\int \pm \frac{\cos x}{\sin x} (dx)}$ or $e^{\int \pm \text{their } P(x)\,(dx)}$	dM1
		$e^{-\ln \sin x}$ or $e^{\ln \operatorname{cosec} x}$	A1 aef
	$= \dfrac{1}{\sin x}$	$\dfrac{1}{\sin x}$ or $(\sin x)^{-1}$ or $\operatorname{cosec} x$	A1 aef
	$\left(\dfrac{1}{\sin x}\right)\dfrac{dy}{dx} - \dfrac{y\cos x}{\sin^2 x} = \dfrac{\sin 2x}{\sin x}$		
	$\dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = \sin 2x \times \dfrac{1}{\sin x}$	$\dfrac{d}{dx}(y \times \text{their I.F.}) = \sin 2x \times \text{their I.F}$	M1
	$\dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = 2\cos x$	$\dfrac{d}{dx}\left(\dfrac{y}{\sin x}\right) = 2\cos x$ or $\dfrac{y}{\sin x} = \int 2\cos x\,(dx)$	A1
	$\dfrac{y}{\sin x} = \int 2\cos x\,dx$		
	$\dfrac{y}{\sin x} = 2\sin x + K$	A credible attempt to integrate the RHS with/without $+K$	dddM1
	$y = 2\sin^2 x + K\sin x$	$y = 2\sin^2 x + K\sin x$	A1 cao
			[8]

Q7.

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx}+2\frac{y}{x}=4x$	M1
	I F: $e^{\int\frac{2}{x}dx}=e^{2\ln x}=(x^2)$	M1
	$x^2\frac{dy}{dx}+2xy=4x^3$	M1dep
	$yx^2=\int 4x^3dx=x^4(+c)$	M1dep
	$y=x^2+\frac{c}{x^2}$	A1cso (5)
(b)	$x=1,\ y=5\Rightarrow c=4$	M1
	$y=x^2+\frac{4}{x^2}$	A1ft (2)
(c)	$\frac{dy}{dx}=2x-\frac{8}{x^3}$	
	$\frac{dy}{dx}=0\quad x^4=4,\ x=\pm\sqrt{2}$ or $\pm\sqrt[4]{4}$	M1,A1
	$y=2+\frac{4}{2}=4$	A1cao
	Alt: Complete square on $y=\ldots$ or use the original differential equation	M1
	$x=\pm\sqrt{2},\quad y=4$	A1,A1
		B1 shape B1 turning points shown *somewhere* (5)
		[12]

Notes for Question

(a)

M1 for dividing the given equation by x May be implied by subsequent work.

M1 for IF= $e^{\int \frac{2}{x}dx} = e^{2\ln x} = (x^2)$ $\int \frac{2}{x}dx$ must be seen together with an attempt at integrating this.

$\ln x$ must be seen in the integrated function.

M1dep for multiplying the equation $\frac{dy}{dx} + 2\frac{y}{x} = 4x$ by their IF dep on 2nd M mark

M1dep for attempting the integration of the resulting equation - constant not needed. Dep on 2nd and 3rd M marks

A1cso for $y = x^2 + \frac{c}{x^2}$ oe eg $yx^2 = x^4 + c$

Alternative: for first three marks: Multiply given equation by x to get straight to the third line. All 3 M marks should be given.

(b)

M1 for using $x = 1,\ y = 5$ in **their** expression for y to obtain a value for c

A1ft for $y = x^2 + \frac{4}{x^2}$ follow through their result from (a)

(c)

M1 for differentiating **their** result from (b), equating to 0 and solving for x

A1 for $x = \pm\sqrt{2}$ (no follow through) or $\pm\sqrt[4]{4}$ No extra real values allowed but ignore any imaginary roots shown.

A1cao for using the particular solution to obtain $y = 4$. No extra values allowed.

Alternatives for these 3 marks:

M1 for making $\frac{dy}{dx} = 0$ in the given differential equation to get $y = 2x^2$ and using this with their particular solution to obtain an equation in one variable

OR complete the square on **their** particular solution to get $y = \left(x + \frac{2}{x}\right)^2 - 4$

A1 for $x = \pm\sqrt{2}$ (no follow through)

A1cao for $y = 4$ No extra values allowed

B1 for the correct shape - must have two minimum points and two branches, both asymptotic to the y-axis

B1 for a fully correct sketch with the coordinates of the minimum points shown somewhere on or beside the sketch. Decimals accepted here.

Q8.

Question Number	Scheme	Marks
(a)	I.F. $= e^{\int 2\tan x\,dx} = e^{2\ln\sec x} = \sec^2 x$	M1A1
	$y\sec^2 x = \int \sec^2 x \sin 2x\,dx$	M1
	$y\sec^2 x = \int \frac{2\sin x\cos x}{\cos^2 x}dx = 2\int \tan x\,dx$	
	$y\sec^2 x = 2\ln\sec x\ (+c)$	M1depA1
	$y = \frac{2\ln\sec x + c}{\sec^2 x}$	A1ft
		(6)
(b)	$y = 2,\ x = \frac{\pi}{3}$	
	$2 = \frac{2\ln\sec\left(\frac{\pi}{3}\right)+c}{\sec^2\left(\frac{\pi}{3}\right)}$	
	$2 = \frac{2\ln(2)+c}{4}$	
	$c = 8 - 2\ln 2$	M1A1
	$x = \frac{\pi}{6}\quad y = \frac{2\ln\sec\left(\frac{\pi}{6}\right)+8-2\ln 2}{\sec^2\left(\frac{\pi}{6}\right)}$	
	$y = \frac{2\ln\frac{2}{\sqrt{3}}+8-2\ln 2}{\frac{4}{3}}$	M1
	$y = \frac{3}{4}\left(8+2\ln\frac{1}{\sqrt{3}}\right) = 6+\frac{3}{2}\ln\frac{1}{\sqrt{3}} = 6-\frac{3}{4}\ln 3$	A1
		(4)
		10 Marks

Question Number	Scheme	Marks
	Alternative: c may not appear explicitly:	
	$y\sec^2\frac{\pi}{6}-2\sec^2\frac{\pi}{3}=2\ln\left(\dfrac{\sec\frac{\pi}{6}}{\sec\frac{\pi}{3}}\right)$	M1A1
	$\frac{4}{3}y-8=2\ln\frac{1}{\sqrt{3}}$	
	$y=\dfrac{3}{4}\left(8+2\ln\dfrac{1}{\sqrt{3}}\right)=6+\dfrac{3}{2}\ln\dfrac{1}{\sqrt{3}}=6-\dfrac{3}{4}\ln 3$	M1A1

NOTES

Question a

M1 for the $e^{\int 2\tan x\,dx}$ or $e^{\int \tan x\,dx}$ and attempting the integration - $e^{(2)\ln\sec x}$ should be seen if final result is not $\sec^2 x$

A1 for IF $=\sec^2 x$

M1 for multiplying the equation by **their** IF and attempting to integrate the lhs

M1dep for attempting the integration of the rhs $\sin 2x=2\sin x\cos x$ and $\sec x=\dfrac{1}{\cos x}$ needed. Dependent on the second M mark

A1cao for all integration correct ie $y\sec^2 x=2\ln\sec x\ (+c)$ constant not needed

A1ft for re-writing **their** answer in the form $y=\ldots$ Accept any equivalent form but the constant must be present. eg $y=\dfrac{\ln\left(A\sec^2 x\right)}{\sec^2 x}$, $y=\cos^2 x\left[\ln\left(\sec^2 x\right)+c\right]$

Notes for Question Continued

Question b

M1 for using the given values $y=2,\ x=\frac{\pi}{3}$ in **their** general solution to obtain a value for the constant of integration

A1 for eg $c=8-2\ln 2$ or $A=\frac{1}{4}e^{8}$ (Check the constant is correct for their correct answer for (a)).

Answers to 3 significant figures acceptable here and can include $\cos\frac{\pi}{3}$ or $\sec\frac{\pi}{3}$

M1 for using **their** constant and $x=\frac{\pi}{6}$ in **their** general solution and attempting the simplification to the required form.

A1cao for $y=6-\frac{3}{4}\ln 3$ $\left(\frac{3}{4}\text{ or }0.75\right)$

Alternative to b

M1 for finding the difference between $y\sec^2\frac{\pi}{6}$ and $2\sec^2\frac{\pi}{3}$ (or equivalent with their general solution)

A1 for $y\sec^2\frac{\pi}{6}-2\sec^2\frac{\pi}{3}=2\ln\left(\frac{\sec\frac{\pi}{6}}{\sec\frac{\pi}{3}}\right)$

M1 for re-arranging to $y=\ldots$ and attempting the simplification to the required form

A1cao for $y=6-\frac{3}{4}\ln 3$ $\left(\frac{3}{4}\text{ or }0.75\right)$

Q9.

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx}+2y\tan x=e^{4x}\cos^2 x$	
	$e^{2\int\tan x\,dx}=e^{2\ln\sec x}=\sec^2 x$ or $\frac{1}{\cos^2 x}$	M1A1
	$\sec^2 x\frac{dy}{dx}+2y\tan x\sec^2 x=e^{4x}\cos^2 x\sec^2 x$	dM1
	$\frac{d}{dx}\left(y\sec^2 x\right)=e^{4x}$	B1ft($y\sec^2 x$)
	$y\sec^2 x=\frac{1}{4}e^{4x}\quad(+c)$	M1
	$y=\left(\frac{1}{4}e^{4x}+c\right)\cos^2 x$ oe	A1
		(6)
(b)	$y=1,\ x=0\quad 1=\left(\frac{1}{4}+c\right)$	M1
	$c=\frac{3}{4}$	
	$y=\frac{1}{4}\left(e^{4x}+3\right)\cos^2 x$ oe	A1
		(2) **[8]**

Notes for Question

(a)	M1 attempting the integrating factor, including integration of (2)$\tan x$ $\ln\cos$ or $\ln\sec$ seen A1 correct integrating factor $\sec^2 x$ or $\frac{1}{\cos^2 x}$ M1 multiplying the equation by the integrating factor – may be implied by the next line. B1ft $y\times$ their IF M1 attempting a complete integration of rhs Must include ke^{4x} but $4e^{4x}$ would imply differentiation. Constant not needed (Incorrect IF may lead to integration by parts, so integration must be complete) A1 correct solution in form $y=\ldots$ constant must be included	
(b)	M1 using given initial conditions to obtain a value for c A1 fully correct final answer May be in the form $y\sec^2 x=\ldots$ or $4y\sec^2 x=\ldots$	

Q10.

Question Number	Scheme	Marks
	$\frac{dy}{dx}+\frac{y}{\tan x}=3\cos 2x$	
	$\int \cot x\,dx=\ln\lvert\sin x\rvert$, IF $=\sin x$	M1
	$\sin x\frac{dy}{dx}+y\cos x=3\cos 2x\sin x$	
	$y\sin x=\int 3\cos 2x\sin x\,dx$	M1A1
	$y\sin x=\int 3\left(2\cos^2 x-1\right)\sin x\,dx$ | $y\sin x=\frac{3}{2}\int (\sin 3x-\sin x)\,dx$	
	$y\sin x=3\left[-\frac{2}{3}\cos^3 x+\cos x\right](+c)$ | $y\sin x=\frac{3}{2}\left[-\frac{1}{3}\cos 3x+\cos x\right](+c)$	dM1A1
	$y=\frac{3\cos x-2\cos^3 x+c'}{\sin x}$ oe | $y=\frac{-3\cos 3x+3\cos x+c'}{2\sin x}$	B1ft [6] (A1 on e-PEN)

M1 Divide by tan and attempt IF $e^{\int \cot x\,dx}$ or equivalent needed

M1 Multiply through by IF and integrate LHS

A1 correct so far

dM1 dep (on previous M mark) integrate RHS using double angle or factor formula

$$k\cos^2 x\sin x\to\pm\cos^3 x,\ k\sin^2 x\cos x\to k\sin^3 x,\ \cos 3x\to\pm\frac{1}{3}\sin 3x,\ \sin 3x\to\pm\frac{1}{3}\cos 3x$$

A1 All correct so far constant not needed

B1ft obtain answer in form $y=\ldots$ any equivalent form Constant must be included and dealt with correctly. Award if correctly obtained from the previous line

Alternatives for integrating the RHS:

(i) By parts: Needs 2 applications of parts or one application followed by a trig method. Give M1 only if method is complete and A1 for a correct result.

(ii) $y\sin x=\int 3\left(1-2\sin^2 x\right)\sin x\,dx=\int 3\sin x-6\sin^3 x\,dx$

Then use $\sin 3x=3\sin x-4\sin^3 x$ to get $y\sin x=\int \frac{3}{2}(\sin 3x-\sin x)\,dx$ and integration shown above - both steps needed for M1

	ALTERNATIVE: Mult through by $\cos x$	
	$\sin x\frac{dy}{dx}+y\cos x=3\cos 2x\sin x$	M1
	$y\sin x=\int 3\cos 2x\sin x\,dx$	M1A1
	Rest as main scheme	

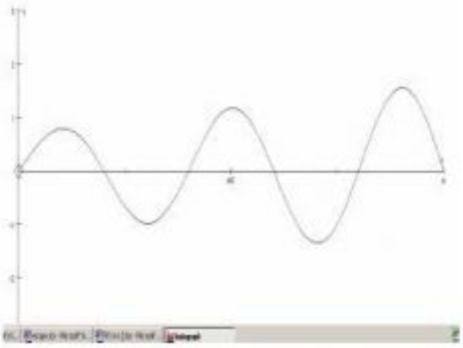
Q11.

Question Number	Scheme	Marks
	$m^2+5m+6=0 \qquad m=-2,\ -3$	M1
	C.F. $(x=)Ae^{-2t}+Be^{-3t}$	A1
	P.I. $x=P\cos t+Q\sin t$ $\dot{x}=-P\sin t+Q\cos t$	B1
	$\ddot{x}=-P\cos t-Q\sin t$	M1
	$(-P\cos t-Q\sin t)+5(-P\sin t+Q\cos t)+6(P\cos t+Q\sin t)=2\cos t-\mathrm{si}$	M1
	$-P+5Q+6P=2$ and $-Q-5P+6Q=-1$, and solve for P and Q	M1
	$P=\frac{3}{10}$ and $Q=\frac{1}{10}$	A1 A1
	$x=Ae^{-2t}+Be^{-3t}+\frac{3}{10}\cos t+\frac{1}{10}\sin t$	B1 ft
		(9) **9**
	Notes 1st M1 form quadratic and attempt to solve (usual rules) 1st B1 Accept negative signs for coefficients. Coefficients must be different. 2nd M1for differentiating their trig PI twice 3rd M1 for substituting x, $\dot{x}$ and $\ddot{x}$ expressions 4th M1 Form 2 equations in two unknowns and attempt to solve 1st A1 for one correct, 2nd A1 for two correct 2nd B1 for x=their CF + their PI as functions of t Condone use of the wrong variable (e.g. x instead of t) for all marks except final B1.	

Q12.

Question Number	Scheme	Marks
(a)	$m^2+6m+9=0 \qquad m=-3$	M1
	C.F. $x=(A+Bt)e^{-3t}$	A1
	P.I. $x=P\cos 3t+Q\sin 3t$	B1
	$\dot{x}=-3P\sin 3t+3Q\cos 3t$ $\ddot{x}=-9P\cos 3t-9Q\sin 3t$	M1
	$(-9P\cos 3t-9Q\sin 3t)+6(-3P\sin 3t+3Q\cos 3t)+9(P\cos 3t+Q\sin 3t)=\cos$	M1
	$-9P+18Q+9P=1$ and $-9Q-18P+9Q=0$	M1
	$P=0$ and $Q=\frac{1}{18}$	A1
	$x=(A+Bt)e^{-3t}+\frac{1}{18}\sin 3t$	A1ft
		(8)
(b)	$t=0$: $x=A=\frac{1}{2}$	B1
	$\dot{x}=-3(A+Bt)e^{-3t}+Be^{-3t}+\frac{3}{18}\cos 3t$	M1
	$t=0$: $\dot{x}=-3A+B+\frac{1}{6}=0 \qquad B=\frac{4}{3}$	M1 A1
	$x=\left(\frac{1}{2}+\frac{4t}{3}\right)e^{-3t}+\frac{1}{18}\sin 3t$	A1
		(5)
(c)	$t\approx\frac{59\pi}{6}$ (≈ 30.9)	B1
	$x\approx-\frac{1}{18}$	B1ft
		(2) **15**
(a) (b)	1st M1 Form auxiliary equation and correct attempt to solve. Can be implied from correct exponential. 2nd M1 for attempt to differentiate PI twice 3rd M1 for substituting their expression into differential equation 4th M1 for substitution of both boundary values 1st M1 for correct attempt to differentiate their answer to part (a) 2nd M1 for substituting boundary value	

Q13.

Question Number	Scheme	Marks
(a)	Differentiate twice and obtaining $\frac{dy}{dx} = \lambda \sin 5x + 5\lambda x \cos 5x$ and $\frac{d^2y}{dx^2} = 10\lambda \cos 5x - 25\lambda x \sin 5x$	M1 A1
	Substitute to give $\lambda = \frac{3}{10}$	M1 A1 (4)
(b)	Complementary function is $y = A\cos 5x + B\sin 5x$ or $Pe^{5ix} + Qe^{-5ix}$	M1 A1
	So general solution is $y = A\cos 5x + B\sin 5x + \frac{3}{10}x\sin 5x$ or in exponential form	A1ft (3)
(c)	$y = 0$ when $x = 0$ means $A = 0$	B1
	$\frac{dy}{dx} = 5B\cos 5x + \frac{3}{10}\sin 5x + \frac{3}{2}x\cos 5x$ and at $x = 0$ $\frac{dy}{dx} = 5$ and so $5 = 5A$	M1 M1
	So $B = 1$	A1
	So $y = \sin 5x + \frac{3}{10}x\sin 5x$	A1 (5)
(d)	 "Sinusoidal" through O amplitude becoming larger Crosses x axis at $\frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$	B1 B1 (2) **14**

Q14.

Question Number	Scheme		Marks
Q	$\dfrac{d^2x}{dt^2} + 5\dfrac{dx}{dt} + 6x = 2e^{-t}$, $x = 0$, $\dfrac{dx}{dt} = 2$ at $t = 0$.		
(a)	AE, $m^2 + 5m + 6 = 0 \Rightarrow (m+3)(m+2) = 0$ $\Rightarrow m = -3, -2.$		
	So, $x_{CF} = Ae^{-3t} + Be^{-2t}$	$Ae^{m_1 t} + Be^{m_2 t}$, where $m_1 \neq m_2$.	M1
		$Ae^{-3t} + Be^{-2t}$	A1
	$\left\{ x = ke^{-t} \Rightarrow \dfrac{dx}{dt} = -ke^{-t} \Rightarrow \dfrac{d^2x}{dt^2} = ke^{-t} \right\}$		
	$\Rightarrow ke^{-t} + 5(-ke^{-t}) + 6ke^{-t} = 2e^{-t} \Rightarrow 2ke^{-t} = 2e^{-t}$	Substitutes ke^{-t} into the differential equation given in the question.	M1
	$\Rightarrow k = 1$	Finds $k = 1$.	A1
	$\left\{ \text{So, } x_{PI} = e^{-t} \right\}$		
	So, $x = Ae^{-3t} + Be^{-2t} + e^{-t}$	their x_{CF} + their x_{PI}	M1*
	$\dfrac{dx}{dt} = = -3Ae^{-3t} - 2Be^{-2t} - e^{-t}$	Finds $\dfrac{dx}{dt}$ by differentiating their x_{CF} and their x_{PI}	dM1*
	$t = 0,\ x = 0 \Rightarrow \quad 0 = A + B + 1$ $t = 0, \dfrac{dx}{dt} = 2 \Rightarrow \quad 2 = -3A - 2B - 1$	Applies $t = 0,\ x = 0$ to x and $t = 0, \dfrac{dx}{dt} = 2$ to $\dfrac{dx}{dt}$ to form simultaneous equations.	ddM1*
	$\begin{Bmatrix} 2A + 2B = -2 \\ -3A - 2B = 3 \end{Bmatrix}$		
	$\Rightarrow A = -1, B = 0$		
	So, $x = -e^{-3t} + e^{-t}$	$x = -e^{-3t} + e^{-t}$	A1 cao
			(8)

Question Number	Scheme		Marks
	$x = -e^{-3t} + e^{-t}$		
(b)	$\frac{dx}{dt} = 3e^{-3t} - e^{-t} = 0$	Differentiates their x to give $\frac{dx}{dt}$ and puts $\frac{dx}{dt}$ equal to 0.	M1
	$3 - e^{2t} = 0$	A credible attempt to solve.	dM1*
	$\Rightarrow t = \frac{1}{2}\ln 3$	$t = \frac{1}{2}\ln 3$ or $t = \ln\sqrt{3}$ or awrt 0.55	A1
	So, $x = -e^{-\frac{3}{2}\ln 3} + e^{-\frac{1}{2}\ln 3} = -e^{\ln 3^{-\frac{3}{2}}} + e^{\ln 3^{-\frac{1}{2}}}$ $x = -3^{-\frac{3}{2}} + 3^{-\frac{1}{2}}$	Substitutes their t back into x and an attempt to eliminate out the ln's.	ddM1
	$= -\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} = \underline{\frac{2\sqrt{3}}{9}}$	uses exact values to give $\frac{2\sqrt{3}}{9}$	A1 AG
	$\frac{d^2x}{dt^2} = -9e^{-3t} + e^{-t}$ At $t = \frac{1}{2}\ln 3$, $\frac{d^2x}{dt^2} = -9e^{-\frac{3}{2}\ln 3} + e^{-\frac{1}{2}\ln 3}$	Finds $\frac{d^2x}{dt^2}$ and substitutes their t into $\frac{d^2x}{dt^2}$	dM1*
	$= -9(3)^{-\frac{3}{2}} + 3^{-\frac{1}{2}} = -\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = -\frac{3}{\sqrt{3}} + \frac{1}{\sqrt{3}}$		
	As $\frac{d^2x}{dt^2} = -\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \left\{-\frac{2}{\sqrt{3}}\right\} < 0$ then x is maximum.	$-\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} < 0$ and maximum conclusion.	A1
			(7)
			[15]

Q15.

Question Number	Scheme	Marks	
(a)	$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$	M1	
	$\dfrac{d^2y}{dx^2} = \dfrac{dv}{dx} + \dfrac{dv}{dx} + x\dfrac{d^2v}{dx^2}$	M1A1	
	$4x^2\left(2\dfrac{dv}{dx} + x\dfrac{d^2v}{dx^2}\right) - 8x\left(v + x\dfrac{dv}{dx}\right) + \left(8+4x^2\right)\times xv = x^4$	M1	
	$4x^3\dfrac{d^2v}{dx^2} + 4x^3v = x^4$	M1	
	$4\dfrac{d^2v}{dx^2} + 4v = x$ *	A1	(6)
	See end for an alternative for (a)		
(b)	$4\lambda^2 + 4 = 0$		
	$\lambda^2 = -1$ oe	M1A1	
	$(v=)C\cos x + D\sin x \quad \left(\text{or } (v=)Ae^{ix} + Be^{-ix}\right)$	A1	
	P.I: Try $v = kx\ (+l)$		
	$\dfrac{dv}{dx} = k \quad \dfrac{d^2v}{dx^2} = 0$	M1	
	$4\times 0 + 4\left(kx\ (+l)\right) = x$	M1dep	
	$k = \dfrac{1}{4} \quad (l = 0)$		
	$v = C\cos x + D\sin x + \dfrac{1}{4}x \quad \left(\text{or } v = Ae^{ix} + Be^{-ix} + \dfrac{1}{4}x\right)$	A1	(6)
(c)	$y = x\left(C\cos x + D\sin x + \dfrac{1}{4}x\right) \quad \left(\text{or } y = x\left(Ae^{ix} + Be^{-ix} + \dfrac{1}{4}x\right)\right)$	B1ft	(1)

Question continued		
	Alternative for (a):	
	$v = \frac{y}{x}$	
	$\frac{dv}{dx} = \frac{dy}{dx} \times \frac{1}{x} - y \times \frac{1}{x^2}$	M1
	$\frac{d^2v}{dx^2} = \frac{d^2y}{dx^2} \times \frac{1}{x} - \frac{dy}{dx} \times \frac{1}{x^2} - \frac{dy}{dx} \times \frac{1}{x^2} + 2y \times \frac{1}{x^3}$	M1A1
	$x^3 \frac{d^2v}{dx^2} = x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y$	M1
	$4x^3 \frac{d^2v}{dx^2} + 4x^3 v = 4x^2 \frac{d^2y}{dx^2} - 8x\frac{dy}{dx} + 8y + 4x^2 y = x^4$	M1
	$4\frac{d^2v}{dx^2} + 4v = x$ *	A1

Notes for Question

(a)

M1 for attempting to differentiate $y = xv$ to get $\frac{dy}{dx}$ - product rule must be used

M1 for differentiating **their** $\frac{dy}{dx}$ to obtain an expression for $\frac{d^2y}{dx^2}$ - product rule must be used

A1 for $\frac{d^2y}{dx^2} = \frac{dv}{dx} + \frac{dv}{dx} + x\frac{d^2v}{dx^2}$

M1 for substituting **their** $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and $y = xv$ in the original equation to obtain a differential equation in v and x

M1 for collecting the terms to have at most a 4 term equation - 4 terms only if a previous error causes $\frac{dv}{dx}$ to be included, otherwise 3 terms

A1cao and cso for $4\frac{d^2v}{dx^2} + 4v = x$ *

Alternative: (see end of mark scheme)

M1 for writing $v = \frac{y}{x}$ and attempting to differentiate by quotient or product rule to get $\frac{dv}{dx}$

M1 for differentiating **their** $\frac{dv}{dx}$ to obtain an expression for $\frac{d^2v}{dx^2}$ - product or quotient rule must be used

A1 for $\frac{d^2v}{dx^2} = \frac{d^2y}{dx^2} \times \frac{1}{x} - \frac{dy}{dx} \times \frac{1}{x^2} - \frac{dy}{dx} \times \frac{1}{x^2} + 2y \times \frac{1}{x^3}$

M1 for multiplying **their** $\frac{d^2v}{dx^2}$ by x^3

M1 for multiplying by 4 **and** adding $4x^2y$ to each side and equating to x^4 (as rhs is now identical to the original equation.

A1cao and cso for $4\frac{d^2v}{dx^2} + 4v = x$ *

(b)

M1 for forming the auxiliary equation and attempting to solve

A1 for $\lambda^2 = -1$ oe

A1 for the complementary function in either form. Award for a correct CF even if $\lambda = \mathrm{i}$ only is shown.

Notes for Question continued

M1 for trying one of $v = kx,\ k \neq 1$ or $v = kx + l$ and $v = mx^2 + kx + l$ as a PI and obtaining $\frac{\mathrm{d}v}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2v}{\mathrm{d}x^2}$

M1dep for substituting their differentials in the equation $4\frac{\mathrm{d}^2v}{\mathrm{d}x^2} + 4v = x$. Award M0 if the original equation is used. Dep on 2nd M mark of (b)

A1cao for obtaining the correct result (either form)

(c)

B1ft for reversing the substitution to get $y = x\left(C\cos x + D\sin x + \frac{1}{4}x\right)$

$\left(\text{or } y = x\left(Ae^{ix} + Be^{-ix} + \frac{1}{4}x\right)\right)$ follow through their answer to (b)

Q16.

Question Number	Scheme	Marks
(a)	$y = \lambda t^2 e^{3t}$	
	$\dfrac{dy}{dt} = 2\lambda t e^{3t} + 3\lambda t^2 e^{3t}$	M1A1
	$\dfrac{d^2y}{dt^2} = 2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t}$	A1
	$2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t} - 12\lambda t e^{3t} - 18\lambda t^2 e^{3t} + 9\lambda t^2 e^{3t} = 6e^{3t}$	M1dep
	$\lambda = 3$	A1cso
		(5)
	NB. Candidates who give $\lambda = 3$ without all this working get 5/5 provided no erroneous working is seen.	
(b)	$m^2 - 6m + 9 = 0$	
	$(m-3)^2 = 0$	
	C.F. $(y =)$ $(A + Bt)e^{3t}$	M1A1
	G.S. $y = (A + Bt)e^{3t} + 3t^2 e^{3t}$	A1ft
		(3)
(c)	$t = 0$ $y = 5$ $\Rightarrow A = 5$	B1
	$\dfrac{dy}{dt} = Be^{3t} + 3(A + Bt)e^{3t} + 6te^{3t} + 9t^2 e^{3t}$	M1
	$\dfrac{dy}{dt} = 4$ $\quad 4 = B + 15$	M1dep
	$B = -11$	A1
	Solution: $y = (5 - 11t)e^{3t} + 3t^2 e^{3t}$	A1ft
		(5)
		13 Marks

Notes for Question

Question a

M1 for differentiating $y = \lambda t^2 e^{3t}$ wrt t. Product rule must be used.

A1 for correct differentiation ie $\frac{dy}{dt} = 2\lambda t e^{3t} + 3\lambda t^2 e^{3t}$

A1 for a correct second differential $\frac{d^2y}{dt^2} = 2\lambda e^{3t} + 6\lambda t e^{3t} + 6\lambda t e^{3t} + 9\lambda t^2 e^{3t}$

M1dep for substituting their differentials in the equation and obtaining a numerical value for λ
Dependent on the first M mark.

A1cso for $\lambda = 3$ (no incorrect working seen)

NB. Candidates who give $\lambda = 3$ without all this working get 5/5 provided no erroneous working is seen. Candidates who attempt the differentiation should be marked on that. If they then go straight to $\lambda = 3$ without showing the substitution, give M1A1 if differentiation correct and M1A0 otherwise, as the solution is incorrect. If $\lambda \neq 3$ then the M mark is only available if the substitution is shown.

Question b

M1 for solving the 3 term quadratic auxiliary equation to obtain a value or values for m (usual rules for solving a quadratic equation)

A1 for the CF $(y =)$ $(A + Bt)e^{3t}$

A1ft for using **their** CF and **their numerical** value of λ in the particular integral to obtain the general solution $y = (A + Bt)e^{3t} + 3t^2e^{3t}$ Must have $y = ...$ and rhs must be a function of t.

Question c

B1 for deducing that $A = 5$

M1 for differentiating **their** GS to obtain $\frac{dy}{dt} = ...$ The product rule must be used.

M1dep for using $\frac{dy}{dt} = 4$ and **their** value for A in **their** $\frac{dy}{dt}$ to obtain an equation for B Dependent on the previous M mark (of (c))

A1cao and cso for $B = -11$

A1ft for using **their** numerical values A and B in **their** GS from (b) to obtain the particular solution. Must have $y = ...$ and rhs must be a function of t.

Q17.

Question Number	Scheme		Marks
(a)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}+10y=27\mathrm{e}^{-x}$		
	$m^2+2m+10\ (=0)\Rightarrow m=...$	Form and solve the aux equation	M1
	$m=-1\pm 3\mathrm{i}$		A1
	$(y=)\mathrm{e}^{-x}(A\cos 3x+B\sin 3x)$ or $(y=)A\mathrm{e}^{(-1+3\mathrm{i})x}+B\mathrm{e}^{(-1-3\mathrm{i})x}$	$y=$ not needed May be seen with θ instead of x	A1
	$y=k\mathrm{e}^{-x},\ y'=-k\mathrm{e}^{-x},\ y''=k\mathrm{e}^{-x}$	$y=k\mathrm{e}^{-x}$ and attempt to differentiate twice	M1
	$\mathrm{e}^{-x}(k-2k+10k)=27\mathrm{e}^{-x}\Rightarrow k=3$		A1
	$y=\mathrm{e}^{-x}(A\cos 3x+B\sin 3x+3)$ or $y=A\mathrm{e}^{(-1+3\mathrm{i})x}+B\mathrm{e}^{(-1-3\mathrm{i})x}+3\mathrm{e}^{-x}$	Must be x and have $y=$ ....Ignore any attempts to change the second form. (But see note at end about marking (b)) ft, so $y=$ their CF + their PI	B1ft (NB A1 on e-PEN)
			(6)
(b)	$x=0,y=0\Rightarrow A=(-3)$	Uses $x=0,y=0$ in an attempt to find A	M1
	$y'=-\mathrm{e}^{-x}(A\cos 3x+B\sin 3x+3)+\mathrm{e}^{-x}(3B\cos 3x-3A\sin 3x)$	M1: Attempt to differentiate using the product rule, with A or their value of A A1: Correct derivative, with A or their value of A	M1A1
	$x=0,y'=0\Rightarrow B=0$	M1: Uses $x=0,y'=0$ and their value of A in an attempt to find B A1: $B=0$	M1A1
	$y=\mathrm{e}^{-x}(3-3\cos 3x)$ oe	cao and cso	A1 (6)
			Total 12

	Alternative for (b) using	$y = Ae^{(-1+3i)x} + Be^{(-1-3i)x} + 3e^{-x}$	
	$x=0, y=0$ to get an equation in A and B	May come from the real part of their derivative instead	M1
	$y' = (-1+3i)Ae^{(-1+3i)x}$ $+(-1-3i)Be^{(-1-3i)x} - 3e^{-x}$	M1: Attempt differentiation using chain rule A1: Correct differentiation	M1A1
	$x=0, y'=0 \Rightarrow -A-B-3=0$ from real parts and $3A-3B=0$ from imaginary parts So $A=B=-\frac{3}{2}$	M1: Uses $x=0, y'=0$ and equates imaginary parts to obtain a second equation for A and B and attempts to solve their equations A1: $A=B=-\frac{3}{2}$	M1A1
	$y = -\dfrac{3}{2}e^{(-1+3i)x} - \dfrac{3}{2}e^{(-1-3i)x} + 3e^{-x}$	A1: Ignore any attempts to change.	A1

Some may change the second form in (a) before proceeding to (b). If their changed form is correct, all marks for (b) are available; if their changed form is incorrect only M marks are available.

Q18.

Question Number	Scheme	Marks
(a)	$x = e^z$	
	$\frac{dx}{dy} = e^z \frac{dz}{dy}$	M1
	$\frac{dy}{dx} = e^{-z} \frac{dy}{dz}$	A1
	$\frac{d^2y}{dx^2} = -e^{-z}\frac{dz}{dx} \times \frac{dy}{dz} + e^{-z}\frac{d^2y}{dz^2} \times \frac{dz}{dx} = \frac{1}{x^2}\left(-\frac{dy}{dz} + \frac{d^2y}{dz^2}\right)$	M1A1A1
	$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 3\ln x$	
	$x^2\left(-\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x^2}\frac{d^2y}{dz^2}\right) + 2x \times \frac{1}{x}\frac{dy}{dz} - 2y = 3z$	M1
	$\frac{d^2y}{dz^2} + \frac{dy}{dz} - 2y = 3z$	A1 (7)
	Alt: $z = \ln x$	
	$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx} = \frac{1}{x}\frac{dy}{dz}$	M1A1
	$\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x}\frac{d^2y}{dz^2} \times \frac{dz}{dx} = -\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x^2}\frac{d^2y}{dz^2}$	M1A1A1
	$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 3\ln x$	
	$x^2\left(-\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x^2}\frac{d^2y}{dz^2}\right) + 2x \times \frac{1}{x}\frac{dy}{dz} - 2y = 3z$	M1
	$\frac{d^2y}{dz^2} + \frac{dy}{dz} - 2y = 3z$	A1 (7)

Question Number	Scheme	Marks	
(b)	Aux eqn: $m^2+m-2=0$		
	$(m+2)(m-1)=0$		
	$m=-2, 1$	M1A1	
	CF: $y=Ae^{-2z}+Be^{z}$	A1	
	PI: Try $y=az+b$		
	$\frac{dy}{dz}=a \quad \frac{d^2y}{dz^2}=0$		
	$a-2(az+b)=3z$		
	$a=-\frac{3}{2}, \quad b=-\frac{3}{4}$	M1	
	Complete soln: $y=Ae^{-2z}+Be^{z}-\frac{3}{2}z-\frac{3}{4}$	A1A1	(6)
(c)	$y=Ax^{-2}+Bx-\frac{3}{2}\ln x-\frac{3}{4}$	B1 ft	(1)
		[14]	

Notes for Question

(a)	M1 differentiates $x=e^{z}$ wrt y; chain rule must be used A1 correct differentiation M1 differentiates again to obtain $\frac{d^2y}{dx^2}$ A1A1 one mark for each correct term M1 substitutes in the given equation A1cso obtains the required equation ALT: Works with $z=\ln x$; marks awarded as above	
(b)	M1 forms and solves the auxiliary equation A1 both values for m correct - may be implied by their CF A1 correct CF M1 tries a suitable expression for the PF and obtains values for the constants in the PF A1A1 shows the complete solution; one mark for each correct term in the PF	
(c)	B1ft reverses the substitution to obtain the solution in the form $y=$ ... Follow through their complete solution from (b)	

Q19.

Question Number	Scheme	Marks
(a)	$x=\mathrm{e}^u \quad \frac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u$ or $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-u}$ or $\frac{\mathrm{d}x}{\mathrm{d}u}=x$ or $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\times\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-u}\frac{\mathrm{d}y}{\mathrm{d}u}$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\mathrm{e}^{-u}\frac{\mathrm{d}u}{\mathrm{d}x}\frac{\mathrm{d}y}{\mathrm{d}u}+\mathrm{e}^{-u}\frac{\mathrm{d}^2y}{\mathrm{d}u^2}\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-2u}\left(-\frac{\mathrm{d}y}{\mathrm{d}u}+\frac{\mathrm{d}^2y}{\mathrm{d}u^2}\right)$	M1A1
	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-7x\frac{\mathrm{d}y}{\mathrm{d}x}+16y=2\ln x$	
	$\mathrm{e}^{2u}\times\mathrm{e}^{-2u}\left(-\frac{\mathrm{d}y}{\mathrm{d}u}+\frac{\mathrm{d}^2y}{\mathrm{d}u^2}\right)-7\mathrm{e}^u\times\mathrm{e}^{-u}\frac{\mathrm{d}y}{\mathrm{d}u}+16y=2\ln\left(\mathrm{e}^u\right)$	dM1
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2}-8\frac{\mathrm{d}y}{\mathrm{d}u}+16y=2u$ *	A1cso (6)

(a) B1 for $\frac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u$ oe as shown seen explicitly or used

M1 obtaining $\frac{\mathrm{d}y}{\mathrm{d}x}$ using chain rule here or seen later

M1 obtaining $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using product rule (penalise lack of chain rule by the A mark)

A1 a correct expression for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ any equivalent form

dM1 substituting in the equation to eliminate x **Only** u and y now Depends on the 2nd M mark

A1cso obtaining the given result from completely correct work

	ALTERNATIVE 1	
	$x=\mathrm{e}^u \quad \frac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u=x$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}u}=\frac{\mathrm{d}y}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}u}=x\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2}=1\frac{\mathrm{d}x}{\mathrm{d}u}\times\frac{\mathrm{d}y}{\mathrm{d}x}+x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\times\frac{\mathrm{d}x}{\mathrm{d}u}=x\frac{\mathrm{d}y}{\mathrm{d}x}+x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$	M1A1
	$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}^2y}{\mathrm{d}u^2}-\frac{\mathrm{d}y}{\mathrm{d}u}$	
	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}u^2}-\frac{\mathrm{d}y}{\mathrm{d}u}\right)-7x\times\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}u}+16y=2\ln\left(\mathrm{e}^u\right)$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2}-8\frac{\mathrm{d}y}{\mathrm{d}u}+16y=2u$ *	dM1A1cso (6)

B1 As above

M1 obtaining $\frac{\mathrm{d}y}{\mathrm{d}u}$ using chain rule here or seen later

M1 obtaining $\frac{\mathrm{d}^2y}{\mathrm{d}u^2}$ using product rule (penalise lack of chain rule by the A mark)

Question Number	Scheme	Marks

A1 Correct expression for $\frac{\mathrm{d}^2y}{\mathrm{d}u^2}$ any equivalent form

dM1A1cso As main scheme

	ALTERNATIVE 2:	
	$u = \ln x \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}u}$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}u} + \frac{1}{x}\frac{\mathrm{d}^2y}{\mathrm{d}u^2} \times \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}u} + \frac{1}{x^2}\frac{\mathrm{d}^2y}{\mathrm{d}u^2}$	M1A1
	$x^2\left(-\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}u} + \frac{1}{x^2}\frac{\mathrm{d}^2y}{\mathrm{d}u^2}\right) - 7x \times \frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}u} + 16y = 2u$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}u^2} - 8\frac{\mathrm{d}y}{\mathrm{d}u} + 16y = 2u$ *	dM1A1cso

See the notes for the main scheme.

There are also **other solutions** which will appear, either starting from equation II and obtaining equation I, or mixing letters x, y and u until the final stage.
Mark as follows:

B1 as shown in schemes above
M1 obtaining a first derivative with chain rule
M1 obtaining a second derivative with product rule
A1 correct second derivative with 2 or 3 variables present
dM1 Either substitute in equation I or substitute in equation II according to method chosen **and** obtain an equation with only y and u (following sub in eqn I) or with only x and y (following sub in eqn II)
A1cso Obtaining the required result from completely correct work

Question Number	Scheme	Marks
(b)	$m^2-8m+16=0$	
	$(m-4)^2=0 \quad m=4$	M1A1
	(CF =) $(A+Bu)\mathrm{e}^{4u}$	A1
	PI: try $y=au+b$ (or $y=cu^2+au+b$ different derivatives, $c=0$)	
	$\frac{\mathrm{d}y}{\mathrm{d}u}=a \quad \frac{\mathrm{d}^2y}{\mathrm{d}u^2}=0$	M1
	$0-8a+16(au+b)=2u$	
	$a=\frac{1}{8} \quad b=\frac{1}{16}$ oe (decimals must be 0.125 and 0.0625)	dM1A1
	$\therefore y=(A+Bu)\mathrm{e}^{4u}+\frac{1}{8}u+\frac{1}{16}$	B1ft (7)
(c)	$y=(A+B\ln x)x^4+\frac{1}{8}\ln x+\frac{1}{16}$	B1 (1)
		[14]

(b) M1 writing down the correct aux equation and solving to $m=\ldots$ (usual rules)

A1 the correct solution ($m=4$)

A1 the correct CF – can use any (single) variable

M1 using an appropriate PI and finding $\frac{\mathrm{d}y}{\mathrm{d}u}$ **and** $\frac{\mathrm{d}^2y}{\mathrm{d}u^2}$ Use of $y=\lambda u$ scores M0

dM1 substitute in the equation to obtain values for the unknowns Dependent on the second M1

A1 correct unknowns two or three ($c=0$)

B1ft a complete solution, follow through their CF and PI. Must have y = a function of u Allow recovery of incorrect variables.

(c) B1 reverse the substitution to obtain a correct expression for y in terms of x No ft here x^4 or $\mathrm{e}^{4\ln x}$ allowed. Must start $y=\ldots$

Q20.

Question Number	Scheme	Marks
(a)	$x = e^u \quad \frac{dx}{du} = e^u$ or $\frac{du}{dx} = e^{-u}$ or $\frac{dx}{du} = x$ or $\frac{du}{dx} = \frac{1}{x}$	
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^{-u}\frac{dy}{du}$	M1A1
	$\frac{d^2y}{dx^2} = -e^{-u}\frac{du}{dx}\frac{dy}{du} + e^{-u}\frac{d^2y}{du^2}\frac{du}{dx} = e^{-2u}\left(-\frac{dy}{du} + \frac{d^2y}{du^2}\right)$	M1A1
	$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = -x^{-2}$	
	$e^{2u} \times e^{-2u}\left(-\frac{dy}{du} + \frac{d^2y}{du^2}\right) - 2e^u \times e^{-u}\frac{dy}{du} + 2y = -e^{-2u}$	dM1
	$\frac{d^2y}{du^2} - 3\frac{dy}{du} + 2y = -e^{-2u}$ *	A1cso (6)
(a)		
M1	obtaining $\frac{dy}{dx}$ using chain rule here or seen later (may not be shown explicitly but appear in the substitution)	
A1	correct expression for $\frac{dy}{dx}$ any equivalent form (again, may not be seen until substitution)	
M1	obtaining $\frac{d^2y}{dx^2}$ using product rule (penalise lack of chain rule by the A mark)	
A1	a correct expression for $\frac{d^2y}{dx^2}$ any equivalent form	
dM1	substituting in the equation to eliminate x **Only** u and y now Depends on both previous M marks. Substitution must have come from their work	
A1cso	obtaining the given result from completely correct work.	

	ALTERNATIVE 1	
	$x = e^u \quad \frac{dx}{du} = e^u = x$	
	$\frac{dy}{du} = \frac{dy}{dx} \times \frac{dx}{du} = x\frac{dy}{dx}$	M1A1
	$\frac{d^2y}{du^2} = 1\frac{dx}{du} \times \frac{dy}{dx} + x\frac{d^2y}{dx^2} \times \frac{dx}{du} = x\frac{dy}{dx} + x^2\frac{d^2y}{dx^2}$	M1A1
	$x^2\frac{d^2y}{dx^2} = \frac{d^2y}{du^2} - \frac{dy}{du}$	
	$\left(\frac{d^2y}{du^2} - \frac{dy}{du}\right) - 2x \times \frac{1}{x}\frac{dy}{du} + 2y = -x^{-2}$	
	$\frac{d^2y}{du^2} - 3\frac{dy}{du} + 2y = -e^{-2u}$ *	dM1A1cso (6)
M1	obtaining $\frac{dy}{du}$ using chain rule here or seen later	
A1	correct expression for $\frac{dy}{du}$ here or seen later	
M1	obtaining $\frac{d^2y}{du^2}$ using product rule (penalise lack of chain rule by the A mark)	
A1	Correct expression for $\frac{d^2y}{du^2}$ any equivalent form	
dM1A1cso	As main scheme	

	ALTERNATIVE 2:	
	$u = \ln x \quad \frac{du}{dx} = \frac{1}{x}$	
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{x}\frac{dy}{du}$	M1A1
	$\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x}\frac{d^2y}{du^2} \times \frac{du}{dx} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x^2}\frac{d^2y}{du^2}$	M1A1
	$x^2\left(-\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x^2}\frac{d^2y}{du^2}\right) - 2x \times \frac{1}{x}\frac{dy}{du} + 2y = -x^{-2}$	
	$\frac{d^2y}{du^2} - 3\frac{dy}{du} + 2y = -e^{-2u}$ * Depends on both previous M marks	dM1A1cso

	There are also **other solutions** which will appear, either starting from equation II and obtaining equation I, or mixing letters x, y and u until the final stage.	
M1	obtaining a first derivative with chain rule	
A1	correct first derivative	
M1	obtaining a second derivative with product rule (Chain rule errors are penalised through A marks)	
A1	correct second derivative with 2 or 3 variables present	
dM1	Either substitute in equation I or substitute in equation II according to method chosen **AND** obtain an equation with only y and u (following sub in eqn I) or with only x and y (following sub in eqn II)	
A1cso	Obtaining the required result from completely correct work	

Question Number	Scheme	Notes	Marks
(b)	$m^2-3m+2=0 \Rightarrow m=1,2$	M1: Forms AE and attempts to solve to $m=$ ...or values seen in CF A1: Both values correct. May only be seen in the CF	M1A1
	$(\text{CF}=)\,A\text{e}^{u}+B\text{e}^{2u}$	CF correct oe can use any (single) variable	A1
	$y=\lambda\text{e}^{-2u}$		
	$\frac{\text{d}y}{\text{d}u}=-2\lambda\text{e}^{-2u}$ $\frac{\text{d}^2y}{\text{d}u^2}=4\lambda\text{e}^{-2u}$	PI of form $y=\lambda\text{e}^{-2u}$ (or $y=\lambda u\text{e}^{-2u}$ if $m=-2$ is a solution of the aux equation) **and** differentiate PI twice wrt u. Allow with x instead of u	M1
	$4\lambda\text{e}^{-2u}+6\lambda\text{e}^{-2u}+2\lambda\text{e}^{-2u}=-\text{e}^{-2u}$ $\Rightarrow \lambda=-\frac{1}{12}$	dM1 substitute in the equation to obtain value for λ Dependent on the second M1 A1 $\lambda=-\frac{1}{12}$	dM1A1
	$y=A\text{e}^{u}+B\text{e}^{2u}-\frac{1}{12}\text{e}^{-2u}$	A complete solution, follow through their CF and PI. Must have $y=$ a function of u Allow recovery of incorrect variables.	B1ft
			(7)
(c)	$y=Ax+Bx^2-\frac{1}{12x^2}$ Or $y=A\text{e}^{\ln x}+B\text{e}^{2\ln x}-\frac{1}{12\text{e}^{2\ln x}}$	Reverse the substitution to obtain a correct expression for y in terms of x No ft here $\frac{1}{12x^2}$ or $\frac{1}{12}x^{-2}$ Must start $y-$ ...	B1
			(1)
			Total 14