

52 MISC 4

2. (a) Not everyone has a phone / is in the directory \rightarrow Biased
- (b) Only one area and one time chosen \rightarrow Biased
- (c) Satisfactory

Random sample: Each selection is independent and all members of population have an equal chance of selection.

4. Not all letters of the alphabet are equally represented in a dictionary.

Assign numbers 1-26 to letters then use a random number generator.

$$6. \quad \sum x = 3033 \quad \sum x^2 = 306676 \quad n = 30$$

$$\bar{x} = \frac{3033}{30} \quad \sigma^2 = \frac{306676 - 101.1^2}{30} \\ \bar{x} = 101.1 \quad = 1.32\bar{3}$$

$$\hat{\mu} = \bar{x} = \underline{101.1} \quad s^2 = \frac{30 \times 1.32\bar{3}}{29} = \underline{1.369}$$

With bags of 10, bag will be $> 1\text{kg}$ if $\bar{X} > 100\text{g}$

$$\bar{X} \sim N(101.1, \frac{1.369}{10})$$

$$P(\bar{X} > 100) = P(Z < \frac{1.1}{\sqrt{\frac{1.369}{10}}}) = \Phi(2.973) = 99.9\%$$

Almost all bags will be $> 1\text{kg}$

8.

(a) As the field is large the botanist may not be able to throw to all areas - the method is satisfactory if she moves around the field as well as changing direction.

(b) The sample is restricted to points spaced equally, 1 m apart, starting 0.5m from the edge of the field. The technique would be satisfactory for a large field though.

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$$

$$P\left(\mu - \frac{\sigma}{10} < \bar{X} < \mu + \frac{\sigma}{10}\right) \geq 0.975$$

$$P\left(Z < \frac{\mu + \frac{\sigma}{10} - \mu}{\sqrt{\frac{\sigma^2}{n}}}\right) \geq 0.9875$$

Due to symmetry, use just higher end:

$$P\left(Z < \frac{\sqrt{n}}{10}\right) \geq 0.9875$$

$$\Phi^{-1}(0.9875) \leq \frac{\sqrt{n}}{10}$$

$$\sqrt{n} \geq 10 \times 2.24 \quad n \geq \underline{502}$$

$$10. \quad \bar{X} \sim N(1.2, \frac{0.075^2}{50})$$

$$\begin{aligned}
 (a) \quad & P(\bar{X} < 1.18) + P(\bar{X} > 1.22) \\
 & = P\left(Z < \frac{1.018 - 1.2}{\sqrt{\frac{0.075^2}{50}}}\right) + 1 - P\left(Z < \frac{1.22 - 1.2}{\sqrt{\frac{0.075^2}{50}}}\right) \\
 & = \underline{0.0594} \\
 (b) \quad & \bar{Y} \sim N(1.15, \frac{0.075^2}{50}) \\
 & P(1.18 < \bar{Y} < 1.22) \\
 & = P\left(Z < \frac{1.22 - 1.15}{\sqrt{\frac{0.075^2}{50}}}\right) - \cancel{P\left(Z < \frac{1.18 - 1.15}{\sqrt{\frac{0.075^2}{50}}}\right)} \\
 & = \underline{0.00234}
 \end{aligned}$$

$$12. \quad (a) \quad \bar{X} \sim N(50, \frac{64}{n})$$

$$\begin{aligned}
 (b) \quad n=25, \quad & P(\bar{X} > 54) \\
 & = \cancel{1 - P\left(z < \frac{54 - 50}{\sqrt{\frac{64}{25}}}\right)} \\
 & = 1 - P\left(z < \frac{54 - 50}{\sqrt{\frac{64}{25}}}\right) \\
 & = 1 - P\left(z < \frac{20}{8}\right) \\
 & = \underline{0.00621}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad & P(\bar{X} > 54) < 0.01 \\
 & P\left(z < \frac{54 - 50}{\sqrt{\frac{64}{n}}}\right) > 0.99
 \end{aligned}$$

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(c) continued

$$\Phi^{-1}(0.99) \leq \frac{\sqrt{n}}{2}$$

$$2.326 \leq \frac{\sqrt{n}}{2}$$

$$(2 \times 2.326)^2 \leq n$$

$$22 \leq n \quad (\text{Nearest integer})$$

When n is large, \bar{x} is approximately normal

When n is small, \bar{x} could be anything